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EXPONENTIAL METHODS OF SUMMATION OF THE FOURIER SERIES

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Abstract: We consider the semi-continuous methods $\Lambda = \{\lambda_k(h),\ k = 0,1,...; h > 0\}$ of summation of Fourier series and conjugated Fourier series, generated by exponential functions $\lambda(x,h) = \exp(-hu^{\alpha}(x)),\ \alpha > 0.$ The estimates of L^p-norms of the corresponding maximal operators are obtained. As consequence, we get some results about exponential method of summation of the Fourier series almost everywhere and in L^p-metric.

Introduction

Consider $f = f(x) \in L([-\pi,\pi]),$ and let

$$U_h(f) = U(f,x;\lambda,h) = \sum_{k=-\infty}^{\infty} \lambda_{|k|}(h) c_k(f) \exp(ikx), \tag{1}$$

$$\widetilde{U}_h(f) = \widetilde{U}(f,x;\lambda,h) = -i \sum_{k=-\infty}^{\infty} (\operatorname{sgn} k) \lambda_{|k|}(h) c_k(f) \exp(ikx) \tag{2}$$

be the set of a linear means of Fourier series and conjugate Fourier series respectively.

In various questions of the analysis there is a problem of behaviour of (1) and (2) when $h \to +0.$ Here $c_k(f) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \exp(-ikt)\, dt, \quad k = 0, \pm 1, \pm 2, ...$ are complex Fourier coefficients,

$$\Lambda = \{\lambda_k(h), k = 0,1,...\} \tag{3}$$

is infinite sequence defined by the values of parameter $h > 0.$ This sequence defines so-called semi-continuous method of summation. The regularity conditions of such methods will be the following [1, p. 79]:

$$\lambda_0(h) = 1,\ \lim_{h\to 0} \lambda_k(h) = 1,\ k = 0,1,...; \tag{4}$$

$$\sup_{h>0} \sum_{k=0}^{\infty} |\Delta\, \lambda_k(h)| < \infty. \tag{5}$$

The similar problems for (1) have been studied by L. I. Bausov [2] in case of discrete h.

We consider the semi-continuous methods of summation corresponding, basically, to the case of

$$\lambda_0(h)=1,\ \ \lambda_k(h)=\lambda(x,h)|_{x=k},\ \ k=1,2,...,$$

where

$$\lambda(x,h)=\exp(-h\varphi(x)), \tag{6}$$

and function $\varphi(x)\in C^2(0,+\infty)$. Note that if $\lambda_k(h)=\exp(-hk)$ we get Poisson-Abel means [3, vol. 1, p. 160 – 165].

Let $\|f\|_p=\left(\int\limits_{-\pi}^{\pi}|f(x)|^p dx\right)^{1/p}$ be a norm in Lebesgue space L^p $\left(p>0;\ L=L^1;\ \|f\|=\|f\|_1\right)$ and

$$\widetilde{f}(x)=-\frac{1}{2}\lim_{\varepsilon\to+0}\int\limits_{\varepsilon\le|t|\le\pi} f(x+t)\operatorname{ctg}\frac{t}{2}dt$$

be a conjugate function; this function exists almost everywhere for each $f\in L$ [3, vol. 1, p. 402]. Define

$$U_*(f)=U_*(f,x;\lambda)=\sup_{h>0}|U\ \ (f,x;\lambda,h)|;\ \ \widetilde{U}_*(f)=\widetilde{U}_*(f,x;\lambda)=\sup_{h>0}|\widetilde{U}(f,x;\lambda,h)|.$$

Estimates of L^p-norms

The sequence (3) is called a convex (concave), if $\Delta_k^2=\Delta(\Delta\lambda_k(h))=\lambda_k(h)-2\lambda_{k+1}(h)+\lambda_{k+2}(h)\ge 0$ $\left(\Delta_k^2\le 0\right)$, $k=0,1,\ldots$ The sequence (3) is piecewise-convex, if Δ_k^2 changes sign a finite number of times, $k=0,1,...$

Theorem 1. If the sequence (3) is a convex (a concave) and

$$\lambda_k(h)\ln k=O(1),\ k\to\infty, \tag{7}$$

for each $h>0$ then the estimates

$$\|U_*(f)\|_p+\|\widetilde{U}_*(f)\|_p\le C_{p,\Lambda}\|(f)\|_p,\quad p>1; \tag{8}$$

$$\|U_*(f)\|\ \ +\|\widetilde{U}_*(f)\|\ \ \le C_\Lambda(1+\|f(\ln^+|f|)\|); \tag{9}$$

$$\|U_*(f)\|_p+\|\widetilde{U}_*(f)\|_p\le C_{p,\Lambda}\|(f)\|,\quad 0<p<1 \tag{10}$$

hold.

Here C will represent a constant, though not necessarily one such constant.

The estimates (8) – (10) remain valid, if a piecewise-convex sequence (3) satisfies to the condition (7) and there is constant $C=C_\Lambda$, such, that

$$|\lambda_k(h)|+k|\Delta\lambda_k(h)|\le C_\Lambda \tag{11}$$

for all $h>0,\ k=1,2,...$

Proofs of both statements will be based on the Abel transform of sums (1), (2) and on the estimates of Fejér means [3, vol. 1, p. 148] by maximal operators

$$f^*=f^*(x)=\sup_{h>0}\frac{1}{h}\int\limits_{x-h}^{x+h}|f(t)|dt\quad\text{and}\quad f^*=f^*(x)=\sup_{h>0}\left|\int\limits_{h\le|t|\le\pi}\frac{f(x+t)}{2\operatorname{tg}\frac{t}{2}}dt\right|.$$

Thus the inequalities (8) – (10) occur when $\| U_*(f) \|_p + \| \tilde{U}_*(f) \|_p$ is replaced by $\| f^* \|_p + \| \tilde{f}^* \|_p$, [1, vol. 1, p. 58–59, 404].

Theorem 2. Let the sequence (3) be a convex (a concave) and at every $h > 0$ satisfies the conditions (7) and (4). Then relations:

$$\lim_{h \to 0} U_h(f) = f; \tag{12}$$

$$\lim_{h \to 0} \tilde{U}_h(f) = \tilde{f} \tag{13}$$

hold almost everywhere (a.e.) for every $f \in L$ and in the metrics L^p for every $p > 1$.

The statements remain valid for every piecewise-convex sequence (3), satisfying to conditions (4), (7), (11).

Besides, under the formulated conditions the relation (12) holds in each point of a continuity of function *f*. The relation (12) holds uniformly over *x* for everyone continuous *f*. Last statement does not extend, generally speaking, on a case (13).

Auxiliary statements

Lemma. If a piecewise-convex sequence (3) satisfies to conditions (7) then the following relations hold:

$$U_*(f, x; \lambda) \le C f^*(x) \sum_{k=0}^{\infty} (k+1) \left| \Delta^2 \lambda_k(h) \right|; \tag{14}$$

$$\tilde{U}_*(f, x; \lambda) \le C \left(f^*(x) + \tilde{f}^*(x) \right) \sum_{k=0}^{\infty} (k+1) \left| \Delta^2 \lambda_k(h) \right|. \tag{15}$$

Proof. We shall prove (15); the relation (14) one can deduce exactly in a similar way. According to the integrated form of Fourier coefficients and Abel transform we obtain

$$\tilde{U}_h(f) = \tilde{U}(f, x; \lambda, h) = \lim_{N \to +\infty} \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \left\{ \sum_{k=1}^{N} \lambda_k(h) \sin k(x-t) \right\} dt =$$

$$= \frac{1}{\pi} \lim_{N \to +\infty} \left\{ \lambda_N(h) \int_{-\pi}^{\pi} f(x+t) \tilde{D}_N(t)\, dt + N \Delta \lambda_{N-1}(h) + \int_{-\pi}^{\pi} f(x+t) \tilde{F}_{N-1}(t)\, dt + \right.$$

$$\left. + \sum_{k=0}^{N-2} (k+1) \Delta^2 \lambda_k(h) \int_{-\pi}^{\pi} f(x+t) \tilde{F}_k(t)\, dt \right\}. \tag{16}$$

Here

$$\tilde{D}_k(t) = \sum_{\nu=1}^{k} \sin \nu t = \frac{1}{2 \operatorname{tg} \frac{1}{2} t} - \frac{\cos\left(k + \frac{1}{2}\right) t}{2 \sin \frac{1}{2} t}; \qquad \tilde{F}_k(t) = \frac{1}{k+1} \sum_{\nu=0}^{k} \tilde{D}(t) = \frac{1}{2 \operatorname{tg} \frac{1}{2} t} - \tilde{\tilde{F}}_k(t)$$

are conjugate Dirichlet and Fejer kernels, respectively, and

$$\tilde{\tilde{F}}_k(t) = \frac{\cos(k+1)t}{2(k+1) \sin^2 \frac{1}{2} t}.$$

Further, we shall establish the following estimates for the integrals containing in the right part of (16):

$$\left|\int_{-\pi}^{\pi} f(x+t)\widetilde{D}_k(t)\,dt\right| \le C f^*(x)\ln k,\ \ k=2,3,...; \tag{17}$$

$$\left|\int_{-\pi}^{\pi} f(x+t)\widetilde{F}_k(t)\,dt\right| \le C\left(f^*(x)+\widetilde{f}^*(x)\right), \quad k=0,1,... \tag{18}$$

For this purpose we shall use the obvious inequalities:

$$\left|\widetilde{D}_k(t)\right|+\left|\widetilde{F}_k(t)\right| \le C(k+1), \quad 0\le t\le \pi, \quad k=0,1,\ldots; \tag{19}$$

$$\left|\widetilde{D}_k(t)\right| \le C\frac{1}{|t|}, \ \ 0<|t|\le\pi, \qquad k=0,1,...; \tag{20}$$

$$\left|\widetilde{\widetilde{F}}_k(t)\right| \le C\frac{1}{(k+1)t^2}, \qquad 0<|t|\le\pi, \qquad k=0,1,... \tag{21}$$

and choose a natural number $S=S(k),\ k=0,1,...,$ such that $\dfrac{2^{S-1}}{k+1}\le\pi<\dfrac{2^S}{k+1}.$ According to (19), (20), we have

$$\left|\int_{-\pi}^{\pi} f(x+t)\widetilde{D}_k(t)\,dt\right| \le C\left((k+1)\int\limits_{|t|\le\frac{1}{k+1}} |f(x+t)|\,dt+\sum_{j=1}^{S}\frac{k+1}{2^{j-1}}\int\limits_{\frac{2^{j-1}}{k+1}\le t\le\frac{2^j}{k+1}}|f(x+t)|\,dt\right)\le$$

$$\le C(1+2S)f^*(x)\le Cf^*(x)\ln k,\ \ k=2,3,...$$

Further, we shall prove (18). In view of relations (19) and (21) we obtain

$$\left|\int_{-\pi}^{\pi} f(x+t)\widetilde{F}_k(t)\,dt\right| \le C\left((k+1)\int\limits_{|t|\le\frac{1}{k+1}}|f(x+t)|\,dt+\left|\int\limits_{\frac{1}{k+1}\le|t|\le\pi}\frac{f(x+t)}{2\operatorname{tg}\frac{t}{2}}dt\right|+\right.$$

$$\left.+\sum_{j=1}^{S}\frac{k+1}{(2^{j-1})^2}\int\limits_{\frac{2^{j-1}}{k+1}\le t\le\frac{2^j}{k+1}}|f(x+t)|\,dt\right)\le C(f^*(x)+\widetilde{f}^*(x)).$$

The statement (15) is now a direct consequence of equality (16), estimates (17), (18), relations (7) and $\Delta\lambda_k(h)=O\left(\dfrac{1}{k}\right),\ k\to\infty.$ The last relation is valid [3, vol. 1, p. 156] for any convex or piecewise-convex sequence.

Proof of the theorems 1, 2

We consider a case of a piecewise-convex sequence Λ. Then $\Delta^2\lambda_k(h)$ keeps the sign for $m\le k\le n$, where m and n are some natural numbers. By Abel summation formula we obtain

$$\sum_{k=m}^{n}(k+1)\Delta^2\lambda_k(h)=\lambda_{m+1}(h)-\lambda_{n+1}(h)+(m+1)\Delta\lambda_m(h)-(n+1)\Delta\lambda_{n+1}(h). \tag{22}$$

Hence, $\sum_{k=0}^{\infty}(k+1)\left|\Delta^2\lambda_k(h)\right|$ is equal to finite number of sums, each of which looks like (22); if $n \to +\infty$ then $\lambda_{n+1}(h)+(n+1)\Delta\lambda_{n+1}(h) \to 0$ [3, vol. 1, p. 155–156]. Now, using relations (11) (14), (15), and (22) we obtain the second statement of the *theorem 1*; the first statement can be received by similar arguments.

The statements of the *theorem 2* (convergence a.e. and in metrics L^p) follow from (22) and (11) by standard arguments [3, vol. 2, p. 464–465]. It is necessary to notice, that the conditions of regularity of Λ -method are valid; the validity of (5) follows [4, p. 748] from (22) and (11).

The convergence (12) in points of continuity and uniformly over x follows from (14) and Banach-Shteinhaus theorem. To use this theorem it is enough obtain the boundedness of Lebesgue constants of summation method. In turn, it will be follow from (14), (22), (11) for $f \equiv 1$ if to notice that $F_k(t) \ge 0$.

The last statement cannot be extended to a case (13) because the conjugate function $\widetilde{f}$ can lose a continuity in points of continuity f [5, p. 554].

Convex and piecewise-convex exponential summarising sequences

We shall address to consideration of a case (6). It be required to us

$$\lambda'_x(x,h) = -h\exp(-h\varphi(x))\varphi'(x),\ \ \lambda''_{xx}(x,h) = h\exp(-h\varphi(x))(h(\varphi'(x))^2 - \varphi''(x)). \quad (23)$$

Let restrict oneself, basically, to consideration of functions

$$\varphi(x) = u^{\alpha}(x),\ \alpha > 0.$$

Theorem 3. Let $u \in C^2(0,+\infty),\ u > 0,\ u'' < 0\ (x \in (0,+\infty)),\quad 0 < \alpha \le 1,$

$$\lambda(x,h) = \exp(-hu^{\alpha}(x)), \quad (24)$$

and

$$\exp(-hu^{\alpha}(x))\ \ln x = O(1),\ \ x \to +\infty \quad (25)$$

for every $h > 0$. Then the estimates (8) – (10) are valid and the relations (12), (13) hold a.e. for every $f \in L$ and in the metrics L^p, $p > 1$. These assertions remain valid if

$$V = V(x) = \alpha h u^{\alpha}(u')^2 - (\alpha-1)(u')^2 - uu'',\quad \alpha > 0 \quad (26)$$

has on $(0,+\infty)$ finite number of zeros, the conditions (25) is satisfied and there is constant $C = C_{u,\alpha}$, such that for all $h > 0,\ x \in (1,+\infty)$

$$xh\exp(-hu^{\alpha}(x))u^{\alpha-1}(x)\left|u'(x)\right| \le C_{u,\alpha}. \quad (27)$$

Proof. We shall apply the results of *theorems 1*, *2*. The condition (7) is satisfied by (25). It is remain to prove that (24) is convex for $0 < \alpha \le 1$. According to (24), (23), (26) we have

$$\lambda''_{xx}(x,h) = \alpha h\exp(-hu^{\alpha}(x))u^{\alpha-2}(x)V(x). \quad (28)$$

Then $\lambda''_{xx}(x,h) < 0$ for $u''(x) < 0$ and $0 < \alpha \le 1$ as it was required to obtain.

Further we shall notice that the formulated condition on function $V(x)$ in (26) provides a piecewise-convexity of sequence (6), defined by (24). Really, let, for example, $V(x)$ is a function of constant sign for $m \le x \le n+2$ (m and n are some non-negative integers). We shall apply to $\lambda(x,h)$ (as functions of x) the Lagrange theorem twice (on $[k, k+1]$ and on $[k+\theta_1, k+2]$ respectively):

$$\Delta\lambda_k(h) = -\lambda'_x(k+\theta_1, h);$$

$$\Delta^2\lambda_k(h) = (1-\theta_1)\lambda''_{xx}(k+\theta_1+\theta_2, h), \tag{29}$$

where $\theta_1 = \theta_1(k)$, $\theta_2 = \theta_2(k)$, $\theta_1, \theta_2 \in (0,1)$. Let $\theta = \theta_1 + \theta_2$. If $m \le k \le n$, then $m < k + \theta < n+2$, such that $\Delta^2\lambda_k(h)$ is sequence of constant sign by (29).

Since a number of intervals (with the integer ends) on which $V(x)$ is a function of constant sign, is finite, then $\Delta^2\lambda_k(h)$ has finite number of changes of a sign. It remain to note the validity of (11), if condition (28) holds.

Theorem 3 is proved.

Examples

1. Let $u(x) = \ln x$, then

$$\lambda_0(h) = 1,\ \lambda(x,h) = \exp\left(-h\ln^\alpha x\right),\ x>0,\ \alpha>0. \tag{30}$$

It is evidently that (25) holds. For $0 < \alpha \le 1$ the sequence (6), defined by (30), is convex.

If $\alpha > 1$, then function (26) vanishes once; hence, the summarising sequence is piece-convex. It is remain to note that $h\exp\left(-h\ln^\alpha x\right)\ln^{\alpha-1} x = \frac{1}{\ln x}$ $\left(h\ln^\alpha x\right)\exp\left(-h\ln^\alpha x\right) \le C_\alpha$ at all $\alpha > 1$ and $x > 2$.

So, the statements of *theorem 3* hold for a case (30) at all $\alpha > 0$. In particular $(\alpha = 1)$

$$c_0(f) + \sum_{1\le|k|<\infty} \frac{1}{k^h} c_k(f)\exp(ikx) \to f(x) \text{ and } -i\sum_{k=-\infty}^{\infty}(\operatorname{sgn} k)\frac{1}{k^h}c_k(f)\exp(ikx) \to \tilde{f}(x)$$

a.e. for everyone $f \in L$ and in metrics L^p, $p > 1$.

2. Let $u(x) = x$, then

$$\lambda_0(h) = 1,\ \lambda(x,h) = \exp\left(-hx^\alpha\right),\ x>0,\ \alpha>0. \tag{31}$$

It is evidently that (25) holds. For $0 < \alpha \le 1$ [6] the sequence (6), defined by (31), is onvex. If $\alpha > 1$, then function (26) vanishes once; hence, the summarising sequence is piece-convex. It is remain to note that

$$h\exp\left(-hx^\alpha\right)x^\alpha \le C_\alpha \text{ at all } \alpha>1 \text{ and } x>0.$$

So, the statements of *theorem 3* hold for a case (31) at all $\alpha > 0$. In particular $\left(\alpha = 1,\ h = \ln\frac{1}{r},\ 0<r<1\right)$ we obtain the convergence of Poisson-Abel means

$$U_r(f,x) = \sum_{k=-\infty}^{\infty} r^{|k|}c_k(f)\exp(ikx) \text{ and } \tilde{U}_r(f,x) = -i\sum_{k=-\infty}^{\infty}(\operatorname{sgn} k)r^{|k|}c_k(f)\exp(ikx).$$

3. Consider a method of summation defined by the function

$$\lambda(x,h)=\exp(-h\,P_n(x)),\ x>0, \tag{32}$$

where $P_n(x)=a_nx^n+a_{n-1}x^{n-1}+...+a_0,\quad a=a_n>0$ is any polynomial, $n=1,2,...$ By relation (23) $\lambda''_{xx}\ (h,x)=\exp(-hP_n(x))h\Psi(h,x)$, where

$$\Psi(h,x)=\ h(P_n'(x))^2-P_n''(x). \tag{33}$$

The right hand part of (33) is a polynomial of degree of $(2n-2)$, so it has no more $(2n-2)$ changes of signs. Hence, the condition of piecewise-convexity of sequence (6) is satisfied.

Verify a condition (11). The production $k|\Delta\lambda_k(h)|$ is a value of function

$$\pi(x)=\frac{xhP_n'(x)}{\exp(hP_n(x))}=\frac{hP_n(x)}{\exp(hP_n(x))}\frac{Q_n(x)}{P_n(x)},$$

where

$$Q_n(x)=xP_n'(x). \tag{34}$$

Then $|\pi(x)|$ is bounded, since $\dfrac{Q_n(x)}{P_n(x)}\to n\quad(x\to+\infty)$, and the first fraction $\dfrac{hP_n(x)}{\exp(hP_n(x))}$ in (34) looks like $\dfrac{t}{\exp t}$, $t>0$.

So, the statements of *theorem 3* hold for a case (32) at all $n=1,2,...$

Operator of the translation type

1. Let $f\in L$ and

$$\tau_h(f):f(x)\mapsto f(x+h)\sim\sum_{k=-\infty}^{\infty}c_k(f)\exp(ikh)\exp(ikx)\qquad(h>0)$$

is a translation operator. Using the integral form of Fourier coefficients $c_k(f)$ we have

$$f(x+h)\sim\left(\frac{1}{\pi}\int_{-\pi}^{\pi}f(t)\left\{\frac{1}{2}+\sum_{k=1}^{\infty}\cos kh\ \cos k(x-t)\right\}dt-\frac{1}{\pi}\int_{-\pi}^{\pi}f(t)\left\{\sum_{k=1}^{\infty}\sin kh\ \sin k(x-t)\right\}dt\right). \tag{35}$$

By analogy with (35) we will consider two summarising sequences Λ, $\widetilde{\Lambda}$ and operator of translation type

$$\tau_h(f)=\tau_h\left(f;\Lambda,\widetilde{\Lambda};x\right):f(x)\mapsto U(f,x;\lambda,h)-\widetilde{U}(f,x;\widetilde{\lambda},h); \tag{36}$$

denote $\tau_*(f)=\tau_*\left(f;\Lambda,\widetilde{\Lambda};x\right)=\sup\limits_{h>0}\left|\tau_h\left(f;\Lambda,\widetilde{\Lambda};x\right)\right|$.

Applying to (36) *theorems 1*, *2*, we obtain the following statements.

Theorem 4. If the elements of each sequence Λ and $\widetilde{\Lambda}$ satisfy to condition (7) and both sequences have certain character of convexity, then the estimates (8) – (10) hold with replacement $\|U_*(f)\|_p+\|\widetilde{U}_*(f)\|_p$ on $\|\tau_*(f)\|_p$. The statement remains valid for any piecewise-convex sequences Λ and $\widetilde{\Lambda}$ which elements satisfy to conditions of a kind (7), (11).

If the condition (4) is besides, satisfied, then the relation

$$\lim_{h\to 0} \tau_h(f) = f - \widetilde{f}$$

holds almost everywhere for everyone $f \in L$ and in metrics L^p at any $p > 1$.

2. The result of *theorem 4* can be applied to the operator $\tau_h(f)$

$$f(x) \mapsto \tau_h(f; u, \alpha; \omega, \beta; x) = \sum_{k=-\infty}^{\infty} \exp(-hu^{\alpha}(|k|))c_k(f)\exp(ikx) +$$

$$+ i \sum_{k=-\infty}^{\infty} (\operatorname{sgn} k) \exp\left(-\frac{1}{h}\omega^{\beta}(|k|)\right) c_k(f) \exp(ikx).$$

Let $u(x)$ be one of the following functions: $u(x) = \ln x$, or $u(x) = x$, and $\omega(x) = \ln x$ or $\omega(x) = x$. Then for every $\alpha > 0,\ \beta > 0$ the relation

$$\lim_{h\to 0} \tau_h(f; u, \alpha; \omega, \beta; x) = f\ (x)$$

holds a.e. for every $f \in L$ and in the metrics L^p $(p > 1)$.

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Экспоненциальные методы суммирования рядов Фурье

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Ключевые слова и фразы: выпуклые, кусочно-выпуклые экспоненциальные суммирующие последовательности; оценки L^p-норм; сходимость почти всюду.

Аннотация: Рассматриваются полунепрерывные методы $\Lambda = \{\lambda_k(h),\ k = 0,1,...; h > 0\}$ суммирования рядов Фурье и сопряженных рядов Фурье, порожденные экспоненциальными функциями $\lambda(x,h) = \exp\left(-hu^{\alpha}(x)\right)$, $\alpha > 0$. Получены оценки L^p-норм соответствующих максимальных операторов. В качестве следствия приводятся некоторые результаты об экспоненциальных методах суммирования рядов Фурье почти всюду и в метрике L^p.

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Exsponentialmethoden der Summierung der Reihen von Fourier

Zusammenfassung: Es werden die halbununterbrochenen Methoden $\Lambda = \{\lambda_k(h),\ k = 0,1,...; h > 0\}$ der Reihen von Fourier und der verknüpften Reihen von Fourier, die von den Expontionalfunktionen $\lambda(x,h) = \exp(-hu^{\alpha}(x)),\quad \alpha > 0.$ erzeugt wurden. Es sind die Einschätzungen von L^p-Norm der entsprechenden maximalen Operatoren erhaltet.Als Untersuchung werden einige Ergebnisse über die Expontionalmethoden der Summierung der Reihen von Fourier fast überall und in der Metrik L^p gebracht.

Méthodes exponentielles de la summation des séries Fourier

Résumé: Sont examinées les méthodes semi-continues $\Lambda = \{\lambda_k(h),\ k = 0,1,...; h > 0\}$ de la summation des séries de Fourier et des séries conjuguées générées par les fonctions exponentielles $\lambda(x,h) = \exp(-hu^{\alpha}(x)),\ \alpha > 0.$ Sont reçues les estimations L^p-normes des opérateurs correspodants. En qualité de conséquence sont cités les résultats sur les méthodes exponentielles de la summation des séries Fourier presque partout et dans la métrique L^p.

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