EXPONENTIAL METHODS OF SUMMATION
OF THE FOURIER SERIES

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Abstract: We consider the semi-continuous methods
\( \Lambda = \{ \lambda_k(h), k = 0,1,\ldots; h > 0 \} \) of summation of Fourier series and conjugated Fourier series, generated by exponential functions \( \lambda(x,h) = \exp(-hu^\alpha(x)), \alpha > 0 \). The estimates of \( L^p \)-norms of the corresponding maximal operators are obtained. As consequence, we get some results about exponential method of summation of the Fourier series almost everywhere and in \( L^p \)-metric.

Introduction

Consider \( f = f(x) \in L([-\pi, \pi]) \), and let
\[
U_h(f) = U(f, x; \lambda, h) = \sum_{k=-\infty}^{\infty} \lambda_k(h) c_k(f) \exp(ikx),
\]
\[
\bar{U}_h(f) = \bar{U}(f, x; \lambda, h) = -i \sum_{k=-\infty}^{\infty} (\text{sgn} k) \lambda_k(h) c_k(f) \exp(ikx)
\]
be the set of a linear means of Fourier series and conjugate Fourier series respectively.

In various questions of the analysis there is a problem of behaviour of (1) and (2) when \( h \to +0 \). Here \( c_k(f) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) \exp(-ikt) dt \), \( k = 0, \pm 1, \pm 2, \ldots \) are complex Fourier coefficients,
\[
\Lambda = \{ \lambda_k(h), k = 0,1,\ldots \}
\]
is infinite sequence defined by the values of parameter \( h > 0 \). This sequence defines so-called semi-continuous method of summation. The regularity conditions of such methods will be the following \([1, p. 79]\):
\[
\lambda_0(h) = 1, \quad \lim_{h \to 0} \lambda_k(h) = 1, \quad k = 0,1,\ldots;
\]
\[
\sup_{h>0} \sum_{k=0}^{\infty} | \Delta \lambda_k(h) | < \infty.
\]
The similar problems for (1) have been studied by L. I. Bausov [2] in case of discrete \( h \).

We consider the semi-continuous methods of summation corresponding, basically, to the case of

\[ \lambda_0(h) = 1, \quad \lambda_k(h) = \lambda(x, h) |_{x = k}, \quad k = 1, 2, \ldots, \]

where

\[ \lambda(x, h) = \exp(-h\varphi(x)), \quad (6) \]

and function \( \varphi(x) \in C^2(0, +\infty) \). Note that if \( \lambda_k(h) = \exp(-hk) \) we get Poisson-Abel means [3, vol. 1, p. 160 – 165].

Let \( \| f \|_p = \left( \frac{1}{p} \int_\pi^\pi |f(x)|^p \, dx \right)^{1/p} \) be a norm in Lebesgue space \( L^p (\rho > 0; \ L = L^1; \ \| f \| = \| f \|_1) \) and

\[ \tilde{f}(x) = \frac{1}{2} \lim_{\varepsilon \to 0} \int_{\varepsilon \leq |t| \leq \pi} \frac{f(x + t) \cot \frac{t}{2}}{2} \, dt \]

be a conjugate function; this function exists almost everywhere for each \( f \in L \) [3, vol. 1, p. 402]. Define

\[ U_*(f) = U_*(f, x; \lambda) = \sup_{h > 0} \{ U_0(f, x; \lambda, h) \}; \quad \tilde{U}_*(f) = \tilde{U}_*(f, x; \lambda) = \sup_{h > 0} \{ \tilde{U}(f, x; \lambda, h) \}. \]

**Estimates of \( L^p \)-norms**

The sequence (3) is called a convex (concave), if

\[ \Delta_k^2 = \Delta(\Delta_k(h)) = \lambda_k(h) - 2\lambda_{k+1}(h) + \lambda_{k+2}(h) \geq 0 \quad (\Delta_k^2 \leq 0), \quad k = 0, 1, \ldots \]

The sequence (3) is piecewise-convex, if \( \Delta_k^2 \) changes sign a finite number of times, \( k = 0, 1, \ldots \)

**Theorem 1.** If the sequence (3) is a convex (a concave) and

\[ \lambda_0(h) \ln k = O(1), \quad k \to \infty, \quad (7) \]

for each \( h > 0 \) then the estimates

\[ \| U_*(f) \|_p + \| \tilde{U}_*(f) \|_p \leq C_{p, \lambda} \| (f) \|_p, \quad p > 1; \quad (8) \]

\[ \| U_*(f) \|_p + \| \tilde{U}_*(f) \|_p \leq C_{\lambda} (1 + \| f (|n| + |f|) \|); \quad (9) \]

\[ \| U_*(f) \|_p + \| \tilde{U}_*(f) \|_p \leq C_{p, \lambda} \| (f) \|_p, \quad 0 < p < 1 \]

(10)

hold.

Here \( C \) will represent a constant, though not necessarily one such constant.

The estimates (8) – (10) remain valid, if a piecewise-convex sequence (3) satisfies to the condition (7) and there is constant \( C = C_{\lambda} \), such that

\[ |\lambda_k(h)| + k |\Delta_k(h)| \leq C_{\lambda} \]

(11)

for all \( h > 0, \ k = 1, 2, \ldots \).

Proofs of both statements will be based on the Abel transform of sums (1), (2) and on the estimates of Fejér means [3, vol. 1, p. 148] by maximal operators

\[ f^* = f^*(x) = \sup_{h > 0} \frac{1}{h} \int_{x-h}^{x+h} f(t) \, dt \quad \text{and} \quad f^* = f^*(x) = \sup_{h > 0} \int_{h \leq |t| \leq \pi} \frac{f(x + t)}{2 \tg \frac{t}{2}} \, dt. \]
Thus the inequalities (8) – (10) occur when \( \|U_*(f)\|_p + \|\tilde{U}_*(f)\|_p \) is replaced by \( \|f^*\|_p + \|\tilde{f}^*\|_p \), [1, vol. 1, p. 58–59, 404].

**Theorem 2.** Let the sequence (3) be a convex (a concave) and at every \( h > 0 \) satisfies the conditions (7) and (4). Then relations:

\[
\lim_{h \to 0} U_h(f) = f; \quad (12)
\]

\[
\lim_{h \to 0} \tilde{U}_h(f) = \tilde{f}; \quad (13)
\]

hold almost everywhere (a.e.) for every \( f \in L \) and in the metrics \( L^p \) for every \( p > 1 \).

The statements remain valid for every piecewise-convex sequence (3), satisfying to conditions (4), (7), (11).

Besides, under the formulated conditions the relation (12) holds in each point of a continuity of function \( f \). The relation (12) holds uniformly over \( x \) for everyone continuous \( f \). Last statement does not extend, generally speaking, on a case (13).

**Auxiliary statements**

**Lemma.** If a piecewise-convex sequence (3) satisfies to conditions (7) then the following relations hold:

\[
U_*(f, x; \lambda) \leq C f^*(x) \sum_{k=0}^{\infty} (k+1) |\Delta^2 \lambda_k(h)|; \quad (14)
\]

\[
\tilde{U}_*(f, x; \lambda) \leq C \left( f^*(x) + \tilde{f}^*(x) \right) \sum_{k=0}^{\infty} (k+1) |\Delta^2 \lambda_k(h)| \quad (15)
\]

Proof. We shall prove (15); the relation (14) one can deduce exactly in a similar way. According to the integrated form of Fourier coefficients and Abel transform we obtain

\[
\tilde{U}_h(f) = \tilde{U}(f, x; \lambda, h) = \lim_{N \to +\infty} \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \left\{ \sum_{k=1}^{N} \lambda_k(h) \sin k(x-t) \right\} dt =
\]

\[
= \frac{1}{\pi} \lim_{N \to +\infty} \left\{ \lambda_N(h) \int_{-\pi}^{\pi} f(x+t) \tilde{D}_N(t) dt + N \Delta \lambda_{N-1}(h) \int_{-\pi}^{\pi} f(x+t) \tilde{F}_{N-1}(t) dt + \right.
\]

\[
+ \sum_{k=0}^{N-2} (k+1) \Delta^2 \lambda_k(h) \int_{-\pi}^{\pi} f(x+t) \tilde{F}_k(t) dt \right\}. \quad (16)
\]

Here

\[
\tilde{D}_k(t) = \sum_{\nu=1}^{k} \sin \nu t = \frac{1}{2t} \left[ \frac{\cos \left( \frac{k+1}{2} t \right)}{2 \sin \frac{1}{2} t} - \frac{\cos \left( \frac{k+1}{2} t \right)}{2 \sin \frac{1}{2} t} \right]; \quad \tilde{F}_k(t) = \frac{1}{k+1} \sum_{\nu=0}^{k} \tilde{D}_\nu(t) = \frac{1}{2t} \left[ \frac{1}{2} - \frac{1}{2} \right] - \frac{\tilde{F}_k(t)}{2}.
\]

are conjugate Dirichlet and Fejer kernels, respectively, and

\[
\tilde{F}_k(t) = \frac{\cos (k+1) t}{2(k+1) \sin ^2 \frac{1}{2} t}.
\]

Further, we shall establish the following estimates for the integrals containing in the right part of (16):
\[
\int_{-\pi}^{\pi} f(x+t) \tilde{D}_k(t) \, dt \leq C f^{*}(x) \ln k, \quad k = 2, 3, \ldots; \quad (17)
\]
\[
\int_{-\pi}^{\pi} f(x+t) \tilde{F}_k(t) \, dt \leq C \left( f^{*}(x) + \tilde{f}^{*}(x) \right), \quad k = 0, 1, \ldots \quad (18)
\]

For this purpose we shall use the obvious inequalities:
\[
|\tilde{D}_k(t)| + |\tilde{F}_k(t)| \leq C(k+1), \quad 0 \leq t \leq \pi, \quad k = 0, 1, \ldots; \quad (19)
\]
\[
|\tilde{D}_k(t)| \leq C \frac{1}{|t|}, \quad 0 < |t| \leq \pi, \quad k = 0, 1, \ldots; \quad (20)
\]
\[
|\tilde{F}_k(t)| \leq C \frac{1}{(k+1)t^2}, \quad 0 < |t| \leq \pi, \quad k = 0, 1, \ldots \quad (21)
\]

and choose a natural number \( S = S(k), \) \( k = 0, 1, \ldots, \) such that \( \frac{2S-1}{k+1} \leq \pi < \frac{2S}{k+1}. \) According to (19), (20), we have
\[
\int_{-\pi}^{\pi} f(x+t) \tilde{D}_k(t) \, dt \leq C \left( (k+1) \int_{|t| \leq \frac{1}{k+1}} |f(x+t)| \, dt + \sum_{j=0}^{S} \frac{k+1}{2j+1} \int_{\frac{2j}{k+1} \leq |t| \leq \frac{2j}{k+1}} |f(x+t)| \, dt \right) \leq
\]
\[
C(1 + 2S) f^{*}(x) \leq Cf^{*}(x) \ln k, \quad k = 2, 3, \ldots
\]

Further, we shall prove (18). In view of relations (19) and (21) we obtain
\[
\int_{-\pi}^{\pi} f(x+t) \tilde{F}_k(t) \, dt \leq C \left( (k+1) \int_{|t| \leq \frac{1}{k+1}} |f(x+t)| \, dt + \int_{\frac{1}{k+1} \leq |t| \leq \frac{\pi}{2}} \frac{f(x+t)}{2 \tan \frac{t}{2}} \, dt \right)
\]
\[
+ \sum_{j=1}^{S} \frac{k+1}{(2j+1)^2} \int_{\frac{2j-1}{k+1} \leq |t| \leq \frac{2j}{k+1}} |f(x+t)| \, dt \leq C (f^{*}(x) + \tilde{f}^{*}(x)).
\]

The statement (15) is now a direct consequence of equality (16), estimates (17), (18), relations (7) and \( \Delta \lambda_k(h) = O \left( \frac{1}{k} \right), \) \( k \to \infty. \) The last relation is valid [3, vol. 1, p. 156] for any convex or piecewise-convex sequence.

**Proof of the theorems 1, 2**

We consider a case of a piecewise-convex sequence \( \Lambda. \) Then \( \Delta^2 \lambda_k(h) \) keeps the sign for \( m \leq k \leq n, \) where \( m \) and \( n \) are some natural numbers. By Abel summation formula we obtain
\[
\sum_{k=m}^{n} (k+1)^2 \Delta^2 \lambda_k(h) = \lambda_{m+1}(h) - \lambda_{n+1}(h) + (m+1)\Delta \lambda_m(h) - (n+1)\Delta \lambda_{n+1}(h).
\]
(22)
Hence, \( \sum_{k=0}^{\infty} (k+1)\left| \Delta^2 \lambda_k(h) \right| \) is equal to finite number of sums, each of which looks like (22); if \( n \to +\infty \) then \( \lambda_n^{h+1}(h) + (n+1)\Delta \lambda_{n+1}(h) \to 0 \) [3, vol. 1, p. 155–156]. Now, using relations (11), (14), (15), and (22) we obtain the second statement of the \textit{theorem 1}; the first statement can be received by similar arguments.

The statements of the \textit{theorem 2} (convergence a.e. and in metrics \( L^p \)) follow from (22) and (11) by standard arguments [3, vol. 2, p. 464–465]. It is necessary to notice, that the conditions of regularity of \( \Lambda \)-method are valid; the validity of (5) follows [4, p. 748] from (22) and (11).

The convergence (12) in points of continuity and uniformly over \( x \) follows from (14) and Banach-Shteinhaus theorem. To use this theorem it is enough obtain the boundedness of eigenvalues constants of summation method. In turn, it will be follow from (14), (22), (11) for \( f \equiv 1 \) if to notice that \( F_k(t) \geq 0 \).

The last statement cannot be extended to a case (13) because the conjugate function \( \bar{f} \) can lose a continuity in points of continuity \( f \) [5, p. 554].

\textit{Convex and piecewise-convex exponential summarising sequences}

We shall address to consideration of a case (6). It be required to us

\[
\lambda'_x(x,h) = -h \exp(-h \phi(x)) \phi'(x), \quad \lambda''_{xx}(x,h) = h \exp(-h \phi(x)) (h \phi'(x))^2 - \phi''(x).
\] (23)

Let restrict oneself, basically, to consideration of functions

\[
\phi(x) = u^\alpha(x), \quad \alpha > 0.
\]

\textit{Theorem 3.} Let \( u \in C^2(0, +\infty) \), \( u > 0 \), \( u'' < 0 \) (\( x \in (0, +\infty) \)), \( 0 < \alpha \leq 1 \),

\[
\lambda(x,h) = \exp(-hu^\alpha(x)), \quad (24)
\]

and

\[
\exp(-hu^\alpha(x)) \ln x = O(1), \quad x \to +\infty
\]

(25)

for every \( h > 0 \). Then the estimates (8) – (10) are valid and the relations (12), (13) hold a.e. for every \( f \in L \) and in the metrics \( L^p \), \( p > 1 \). These assertions remain valid if

\[
V = V(x) = \alpha h u^\alpha(u')^2 - (\alpha - 1)(u')^2 - uu''', \quad \alpha > 0
\]

(26)

has on \( (0, +\infty) \) finite number of zeros, the conditions (25) is satisfied and there is constant \( C = C_{u,\alpha} \) such that for all \( h > 0 \), \( x \in (1, +\infty) \)

\[
xh \exp(-hu^\alpha(x))u^{\alpha-1}(x)u' \leq C_{u,\alpha}.
\]

(27)

Proof. We shall apply the results of \textit{theorems 1, 2}. The condition (7) is satisfied by (25). It is remain to prove that (24) is convex for \( 0 < \alpha \leq 1 \). According to (24), (23), (26) we have

\[
\lambda''_{xx}(x,h) = \alpha h \exp(-hu^\alpha(x))u^{\alpha-2}(x)V(x).
\]

(28)

Then \( \lambda''_{xx}(x,h) < 0 \) for \( u''(x) < 0 \) and \( 0 < \alpha \leq 1 \) as it was required to obtain.

Further we shall notice that the formulated condition on function \( V(x) \) in (26) provides a piecewise-convexity of sequence (6), defined by (24). Really, let, for example, \( V(x) \) is a function of constant sign for \( m \leq x \leq n+2 \) \( (m \) and \( n \) are some
non-negative integers). We shall apply to \( \lambda(x,h) \) (as functions of \( x \)) the Lagrange theorem twice (on \([k, k+1]\) and on \([k + \theta_1, k + 2]\) respectively):

\[
\Delta \lambda_k (h) = -\lambda'_x (k + \theta_1, h);
\]
\[
\Delta^2 \lambda_k (h) = (1 - \theta_1)\lambda''_{xx} (k + \theta_1 + \theta_2, h), \tag{29}
\]
where \( \theta_1 = \theta_1(k), \theta_2 = \theta_2(k), \theta_1, \theta_2 \in (0,1) \). Let \( \theta = \theta_1 + \theta_2 \). If \( m \leq k \leq n \), then \( m < k + \theta < n + 2 \), such that \( \Delta^2 \lambda_k (h) \) is sequence of constant sign by (29).

Since a number of intervals (with the integer ends) on which \( V(x) \) is a function of constant sign, is finite, then \( \Delta^2 \lambda_k (h) \) has finite number of changes of a sign. It remain to note the validity of (11), if condition (28) holds.

**Theorem 3** is proved.

**Examples**

1. Let \( u(x) = \ln x \), then

\[
\lambda_0(h) = 1, \quad \lambda(x, h) = \exp(-h \ln^\alpha x), \quad x > 0, \quad \alpha > 0. \tag{30}
\]

It is evidently that (25) holds. For \( 0 < \alpha \leq 1 \) the sequence (6), defined by (30), is convex.

If \( \alpha > 1 \), then function (26) vanishes once; hence, the summarising sequence is piece-convex. It is remain to note that

\[
\left( h \ln^\alpha x \right) \exp(-h \ln^\alpha x) \leq C_\alpha \text{ at all } \alpha > 1 \text{ and } x > 2.
\]

So, the statements of *Theorem 3* hold for a case (30) at all \( \alpha > 0 \). In particular (\( \alpha = 1 \))

\[
c_0(f) + \sum_{1 \leq |k| < \infty} \frac{1}{k^\alpha} c_k (f) \exp(ikx) \to f(x) \text{ and } -i \sum_{k=\infty}^{\infty} \left( \text{sgn } k \right) \frac{1}{k^\alpha} c_k (f) \exp(ikx) \to \tilde{f}(x)
\]
a.e. for everyone \( f \in L \) and in metrics \( L^p, \quad p > 1 \).

2. Let \( u(x) = \ln x \), then

\[
\lambda_0(h) = 1, \quad \lambda(x, h) = \exp(-h x^\alpha), \quad x > 0, \quad \alpha > 0. \tag{31}
\]

It is evidently that (25) holds. For \( 0 < \alpha \leq 1 \) [6] the sequence (6), defined by (31), is convex. If \( \alpha > 1 \), then function (26) vanishes once; hence, the summarising sequence is piece-convex. It is remain to note that

\[
h \exp(-h x^\alpha) x^\alpha \leq C_\alpha \text{ at all } \alpha > 1 \text{ and } x > 0.
\]

So, the statements of *Theorem 3* hold for a case (31) at all \( \alpha > 0 \). In particular (\( \alpha = 1, h = \ln \frac{1}{r}, \quad 0 < r < 1 \)) we obtain the convergence of Poisson-Abel means

\[
U_r(f, x) = \sum_{k=-\infty}^{\infty} r^{|k|} c_k (f) \exp(ikx) \text{ and } \tilde{U}_r(f, x) = -i \sum_{k=-\infty}^{\infty} \left( \text{sgn } k \right) r^{|k|} c_k (f) \exp(ikx).
\]
3. Consider a method of summation defined by the function
\[ \lambda(x, h) = \exp(-h P_n(x)), \quad x > 0, \] (32)
where \( P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0, \quad a = a_n > 0 \) is any polynomial, \( n = 1, 2, \ldots \). By relation (23) \( \lambda''_{xx} (h, x) = \exp(-h P_n(x)) h \Psi(h, x) \), where
\[ \Psi(h, x) = h (P'_n(x))^2 - P''_n(x). \] (33)
The right hand part of (33) is a polynomial of degree of \((2n - 2)\), so it has no more \((2n - 2)\) changes of signs. Hence, the condition of piecewise-convexity of sequence (6) is satisfied.

Verify a condition (11). The production \( k |\lambda_{\lambda_k} (h)| \) is a value of function
\[ \pi(x) = \frac{xh P'_n(x)}{\exp(h P_n(x))} = \frac{h P_n(x)}{\exp(h P_n(x))} \frac{Q_n(x)}{P_n(x)}, \]
where
\[ Q_n(x) = x P'_n(x). \] (34)
Then \(|\pi(x)|\) is bounded, since \( \frac{h P_n(x)}{\exp(h P_n(x))} \) looks like \( t \frac{1}{\exp t}, \quad t > 0. \) 

So, the statements of theorem 3 hold for a case (32) at all \( n = 1, 2, \ldots \).

Operator of the translation type

1. Let \( f \in L \) and
\[ \tau_h(f) : f(x) \mapsto f(x + h) \sim \sum_{k=-\infty}^{\infty} c_k(f) \exp(ikh) \exp(ikx) \quad (h > 0) \]
is a translation operator. Using the integral form of Fourier coefficients \( c_k(f) \) we have
\[ f(x + h) \sim \left( \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \frac{1}{2} + \sum_{k=1}^{\infty} \cos k h \cos k(x-t) \right) dt - \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \left( \sum_{k=1}^{\infty} \sin k h \sin k(x-t) \right) dt \]. (35)

By analogy with (35) we will consider two summarising sequences \( \Lambda, \tilde{\Lambda} \) and operator of translation type
\[ \tau_h(f) = \tau_h(f; \Lambda; \tilde{\Lambda}; x) : f(x) \mapsto U(f, x; \Lambda, \tilde{\Lambda}, h) - \tilde{U}(f, x; \Lambda, h); \] (36)
denote \( \tau_* (f) = \tau_* (f; \Lambda, \tilde{\Lambda}; x) = \sup_{h > 0} \tau_h(f; \Lambda, \tilde{\Lambda}; x) \).

Applying to (36) theorems 1, 2, we obtain the following statements.

Theorem 4. If the elements of each sequence \( \Lambda \) and \( \tilde{\Lambda} \) satisfy to condition (7) and both sequences have certain character of convexity, then the
estimates (8) – (10) hold with replacement \( \| U_* (f) \|_\rho + \| \tilde{U}_* (f) \|_\rho \) on \( \| \tau_* (f) \|_\rho \). The statement remains valid for any piecewise-convex sequences \( \Lambda \) and \( \tilde{\Lambda} \) which elements satisfy to conditions of a kind (7), (11).

If the condition (4) is besides, satisfied, then the relation...
\[
\lim_{h \to 0} \tau_h(f) = f - \tilde{f}
\]
holds almost everywhere for everyone \( f \in L \) and in metrics \( L^p \) at any \( p > 1 \).

2. The result of theorem 4 can be applied to the operator \( \tau_h(f) \)
\[
f(x) \mapsto \tau_h(f; u, \alpha; \omega, \beta; x) = \sum_{k=-\infty}^{\infty} \exp(-hu^{\alpha}(|k|))c_k(f)\exp(ikx) + \\
+ i \sum_{k=-\infty}^{\infty} (\text{sgn } k) \exp\left(-\frac{1}{h}\omega^{\beta}(|k|)\right)c_k(f)\exp(ikx).
\]
Let \( u(x) \) be one of the following functions: \( u(x) = \ln x \), or \( u(x) = x \), and \( \omega(x) = \ln x \) or \( \omega(x) = x \). Then for every \( \alpha > 0 \), \( \beta > 0 \) the relation
\[
\lim_{h \to 0} \tau_h(f; u, \alpha; \omega, \beta; x) = f(x)
\]
holds a.e. for every \( f \in L \) and in the metrics \( L^p \) (\( p > 1 \)).

References

Список литературы


Exponentialmethoden der Summierung der Reihen von Fourier

Zusammenfassung: Es werden die halbununterbrochenen Methoden \( \Lambda = \{ \lambda_k(h), k = 0,1,\ldots; h > 0 \} \) der Reihen von Fourier und der verknüpften Reihen von Fourier, die von den Exponentialfunktionen \( \lambda(x, h) = \exp(-hu^\alpha(x)), \alpha > 0 \) erzeugt wurden. Es sind die Einschätzungen von \( L^p \)-Norm der entsprechenden maximalen Operatoren erhalten. Als Untersuchung werden einige Ergebnisse über die Exponentialmethoden der Summierung der Reihen von Fourier fast überall und in der Metrik \( L^p \) gebracht.

Méthodes exponentielles de la summation des séries Fourier

Résumé: Sont examinées les méthodes semi-continues \( \Lambda = \{ \lambda_k(h), k = 0,1,\ldots; h > 0 \} \) de la summation des séries de Fourier et des séries conjuguées générées par les fonctions exponentielles \( \lambda(x, h) = \exp(-hu^\alpha(x)), \alpha > 0 \). Sont reçues les estimations \( L^p \)-normes des opérateurs correspondants. En qualité de conséquence sont cités les résultats sur les méthodes exponentielles de la summation des séries Fourier presque partout et dans la métrique \( L^p \).


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