УДК 612.13.001.575

HEMODYNAMICS MODELING OF THE CARDIOVASCULAR SYSTEM WITH A PULSATING HEART

S.V. Frolov[1], S.V. Sindeev[1], V.A. Lishchuck[2], D.Sh. Gazizova[2]

Department "Biomedical Engineering" (1), TSTU; sergej.frolov@gmail.com;
Bakoulev Scientific Center for Cardiovascular Surgery of the Russian Academy of Medical Sciences (2), Moscow

Represented by a Member of the Editorial Board Professor S.I. Dvoretsky

Key words and phrases: cardiovascular system; hemodynamic; mathematical model.

Abstract: The paper presents a mathematical description of the cardiovascular system model with a pulsating heart. The cardiovascular system is represented as series-connected eight elastic chambers.

Abbreviations:

C – elasticity of the chamber, cm^3 /torr;
c – arbitrary constant of general differential equation solution;
E^{SE}, E^{PE} – rigidity factor for serial/parallel resilient elements of a myocardium, torr;
G – geometrical constant;
h – average wall thickness of the chamber of heart, cm;
K^{SE}, K^{PE} – factor of nonlinearity for a rigidity of serial/collateral resilient elements of a myocardium;
k – pump factor;
L – persistence of blood flow, torr·s^2/cm^3;
l – length of an element, cm;
P – pressure in chamber, torr;
q – blood flow, cm^3/s;
R – hydraulic resistance, torr·s/cm;
r – radius of a chamber, cm;
s – the proportion of contractile filaments in a cross-sectional area of myocardial;
T – duration of heartbeat, s;
T_{sys} – duration of systole, s;
t – time, s;
t_{ed} – time of the end of diastole, s;
t_{es} – time of the end of systole, s;
U – unstrained volume, cm^3;
u – systolic tone of the cardiac chamber, cm^3;
V – volume of i-th chamber, cm^3;
V^0 – unstrained chamber volume, cm^3;
V^{ed} – end-diastolic volume, cm^3;
V^{sv} – stroke volume, cm^3;
ε – relative linear deformation;
η – myocardium coefficient of viscosity, torr·s;
σ^{PE} – pressure in parallel elastic element, torr;
σ^{SE} – pressure in serial elastic element, torr;
χ – coefficient of contraction;
ω – volume of pseudo-chamber, cm^3;
interlinear index: i designation of i-th model chamber.

For diagnostics and therapy of cardiac patients the hardware-software complexes [1] based on mathematical models of the cardiovascular system (**CVS**) have been developed (Fig.1).

The mathematical model of CVS consisting of eight series-connected elements: LH – left heart; AR – arterial reservoir; MB – microcirculatory bed; VS – venous system; RH – right heart; PA – pulmonary artery; CPC – capillaries of the pulmonary

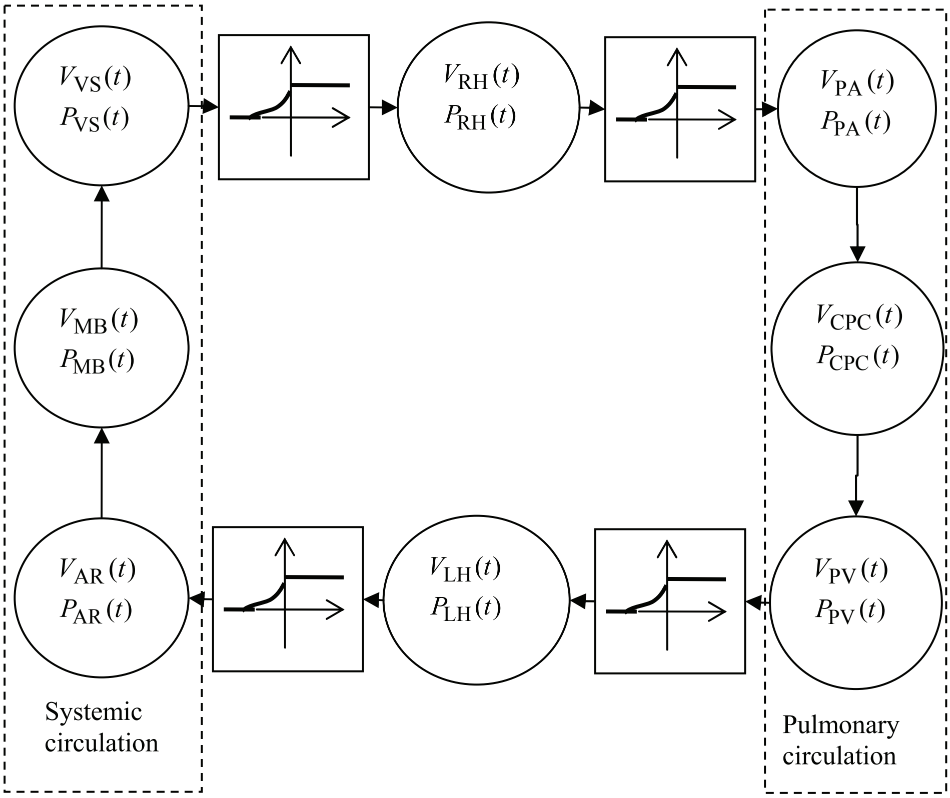

Fig. 1. Chamber structure of blood circulation model

circulation; PV – pulmonary veins. Left heart and Right heart consists of a chamber and a cardiac valve.

Assumptions of CVS model are accepted the same, as in work [2].

In the CVS model, each i-th chamber (Fig. 2) is characterized by the functions of volumetric blood flow, volume, pressure: $q_i(t)$, $V_i(t)$, $Pi(t)$, $i \in \{\mathrm{LH, AR, MB, VS, RH, PA, CPC, PV}\}$.

The change in volume $V_i(t)$ in *i*-th chamber is equal to a difference of inflow $q_{i-1}(t)$ to the chamber and outflow $q_i(t)$ from the chamber; $i \in \{\mathrm{LH, AR, MB, VS, RH, PA, CPC, PV}\}$.

Balance of blood equation in model's chambers differentially looks as follows [2]

$$\frac{dV_i}{dt} = q_{i-1}(t) - q_i(t). \qquad (1)$$

In the modeling of dynamic processes in chambers of systemic and palmonary circulation the inertial properties of blood flow and flow resistance should be taken into account. The right and left chambers of the heart inertia can be neglected, as the blood moves in these cells without

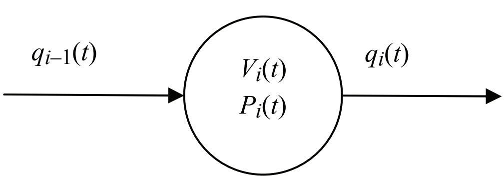

Fig. 2. Chamber of CVS model

any significant acceleration. The equations of motion for the flow are written as follows [2] $i \in \{\mathrm{AR}, \mathrm{MB}, \mathrm{VS}, \mathrm{PA}, \mathrm{CPC}, \mathrm{PV}\}$:

$$q_i(t) = \frac{1}{R_i(t)}\left(P_{i-1}(t) - P_i(t)\right), \quad i \in \{\mathrm{LH}, \mathrm{RH}\}; \tag{2}$$

$$\frac{dq_i(t)}{dt} = \frac{1}{L_i}\left(P_{i-1}(t) - P_i(t) - R_i(t) q_i(t)\right). \tag{3}$$

Resistance R_i, $i \in \{\mathrm{LH}, \mathrm{VS}, \mathrm{RH}, \mathrm{PV}\}$ is determined by the condition of the heart valves. The opening of the valve occurs at a time when there is an arbitrarily small differential pressure flow in the direction of opening. The valve remains fully open until the pressure drop remains in the opening direction of blood flow and, consequently, the direction of blood flow is positive. The closure of the valve is due to displacement (regurgitation) of a certain volume of blood through the valve in the incoming direction opposite to its normal capacity [3]

$$\Delta_i = \begin{cases} 0, \text{ where } \quad q_i(t) > 0; \\ \int\limits_{\tau_1}^{t} q_i(t) dt, \text{ where } \quad q_i(t) \le 0, \end{cases} \tag{4}$$

where τ_i – beginning of the period of time in which blood flow has a negative direction.

With the increase in the volume of regurgitant backflow the resistance will increase from the value R_i^*. Here, R_i^*, $i \in \{\mathrm{LH}, \mathrm{VS}, \mathrm{RH}, \mathrm{PV}\}$ – the hydraulic resistance of the cells. In the presence of blood flow in the opposite direction of the conductivity of the "camera–valve" varies according to the law [5]

$$\rho_i(\Delta_i(t)) = \frac{2}{R_i^*(1 + \exp(-\beta_i \Delta_i(t)))}. \tag{5}$$

Variable resistance in the "camera–valve" is connected with the conductivity

$$\tilde{R}_i(t) = \frac{1}{\rho_i(\Delta_i(t))}. \tag{6}$$

Then, hydraulic resistance of the valves are defined as

$$R_i(t) = \begin{cases} R_i^*, \text{ where } P_i > P_{i+1}; \\ \infty, \text{ where } P_i \le P_{i+1}. \end{cases} \tag{7}$$

The dependence of the pressure function $P_i(t)$ for $i \in \{\mathrm{AR}, \mathrm{MB}, \mathrm{VS}, \mathrm{PA}, \mathrm{CPC}, \mathrm{PV}\}$ on the function of volume $V_i(t)$ is written as an equation of Frank [3, 4]

$$P_i(t) = \frac{1}{C_i}\left(V_i(t) - U_i\right). \tag{8}$$

To determine the dependence of $P_i(t)$ from $V_i(t)$, the model of heart is considered as the 2-chamber tank. For simplicity it is supposed that each chamber of heart is a thin-walled sphere. The dependence of pressure in the chamber is defined by the law of Laplace for a thin-walled sphere, $i \in \{\mathrm{LH}, \mathrm{RH}\}$ [4, 5]

$$\sigma_i(t) = \frac{P_i(t) r_i(t)}{2h_i}; \tag{9}$$

$$r_i(t) = \sqrt[3]{\frac{3V_i(t)}{4\pi}}. \qquad (10)$$

From the equations (9)–(10) it follows:

$$P_i(t) = 4\sqrt[3]{\frac{\pi}{6}}\frac{h}{\sqrt[3]{V_i(t)}}\sigma_i(t), \quad i \in \{\text{LH}, \text{RH}\}. \qquad (11)$$

To determine the function $\sigma_i(t)$ the model of a cardiac muscle is considered. Four-element representation of a cardiac muscle is accepted [6]. The wall of the sphere (chamber) is represented as a cardiac muscle (myocardium).

Pressure $\sigma_i(t)$ in chamber wall is written as follows [5]:

$$\sigma_i(t) = s\sigma_i^{\text{SE}}(t) + (1-s)\sigma_i^{\text{PE}}(t), \quad i \in \{\text{LH}, \text{RH}\}. \qquad (12)$$

Pressure in elastic elements SE, PE of *i*-th chamber is defined as follows [5]:

$$\sigma_i^{\text{SE}}(t) = E_i^{\text{SE}}\left[\exp\left(K_i^{\text{SE}}\varepsilon_i^{\text{SE}}(t)\right) - 1\right]; \qquad (13)$$

$$\sigma_i^{\text{PE}}(t) = E_i^{\text{PE}}\left[\exp\left(K_i^{\text{PE}}\varepsilon_i(t)\right) - 1\right], \qquad (14)$$

where $\varepsilon^{\text{SE}}(t)$ – relative linear deformation of serial elastic element.

From (11) – (14) follows, that pressure in *i*-th chamber is

$$P_i(t) = 4\sqrt[3]{\frac{\pi}{6}}\frac{h_i}{\sqrt[3]{V_i(t)}}\left[E_i^{\text{PE}}(1-s)\left(\exp\left(K_i^{\text{PE}}\varepsilon_i(t)\right) - 1\right) + E_i^{\text{SE}}s\left(\exp\left(K_i^{\text{SE}}\varepsilon_i^{\text{SE}}(t)\right) - 1\right)\right]. \qquad (15)$$

The variables that characterize the linear dimensions of the myocardium strip, are associated with variable $V_i(t)$ and $\omega_i(t)$ [6].

To determine $\omega_i(t)$ in phase of relaxation we consider the rate of change of relative linear deformation of contractile element $\varepsilon_i^{\text{CE}}(t)$, which satisfies the equation [5]:

$$\eta\frac{d\varepsilon_i^{\text{CE}}(t)}{dt} = \sigma_i^{\text{SE}}(t), \quad i \in \{\text{LH}, \text{RH}\}. \qquad (16)$$

Equation (16) corresponds to the four-element representation of the myocardium [6].

As length l_i^0 is equal to sum $l_i^{\text{SE0}} + l_i^{\text{CE0}}$ of $l_i^{\text{CE0}} = l_i^0 - l_i^{\text{SE0}}$, then for the relative linear deformation of the contractile element is obtained $i \in \{\text{LH}, \text{RH}\}$

$$\varepsilon_i^{\text{CE}}(t) = \frac{l_i^{\text{CE}}(t) - l_i^{\text{CE0}}}{l_i^{\text{CE0}}} = \frac{\sqrt[3]{\omega_i(t)} - \sqrt[3]{V_i^0}}{\sqrt[3]{V_i^0} - \sqrt[3]{V_i^{\text{SE0}}}}. \qquad (17)$$

As a result of differentiation (16) in *t* using (13), (17) we obtain formula for the phase relaxation of the myocardium:

$$\frac{d\omega_i(t)}{dt} = \frac{3E_i^{\text{SE}}}{\eta_i}\sqrt[3]{\omega^2{}_i(t)}\left(\sqrt[3]{V_i^0} - \sqrt[3]{V_i^{\text{SE0}}}\right)\left[\exp\left(K_i^{\text{SE}}\varepsilon_i^{\text{SE}}(t) - 1\right)\right], i \in \{\text{LH}, \text{RH}\}. \qquad (18)$$

For the description of a phase of reduction of heart the hypothesis is accepted [4]:

$$\frac{d\omega_i(t)}{dt} = \chi_i\left(\omega_i(t) - u_i\right), \quad i \in \{\text{LH}, \text{RH}\}. \qquad (19)$$

Linear approximation of law is written as follows [4], $i \in \{\mathrm{LH}, \mathrm{RH}\}$

$$V_i^{\mathrm{sv}} = k_i\left(V_i^{\mathrm{ed}} - u_i\right). \qquad (20)$$

In the model of a pulsating heart, the heart activity is seen as an alternation of phases of contraction (systole) and relaxation phase (diastole). The characteristics of this process are: the reduction of heart period T and duration of systole T_{sys}. The beginning of a cardiac cycle is the moment of change of a diastole on a systole. Generally T and T_{sys} can be different for different cycles, then the moments of the systole end $t_{\mathrm{es}}(n)$ and the diastole end of $t_{\mathrm{ed}}(n)$ of n-th n = 1, 2, 3... are expressed by formulas:

$$t_{\mathrm{es}}(n) = \sum_{j=1}^{n-1} T(j) + T_{\mathrm{sys}}(n) + t_0; \qquad (21)$$

$$t_{\mathrm{ed}}(n) = \sum_{j=1}^{n} T(j) + t_0 .$$

Throughout systole the myocardium is subject to equation (19) and throughout diastole it is subject to equation (18). Transition from a systole to a diastole occurs by change of these equations.

Thus, closed system of equations (1)–(8), (15), (18), (19), (21) defines eight-chamber CVS model from which functions of volumes $V_i(t)$, pressure $P_i(t)$ and blood flow $q_i(t)$ in i-th chamber, $i \in \{\mathrm{LH}, \mathrm{AR}, \mathrm{MB}, \mathrm{VS}, \mathrm{RH}, \mathrm{PA}, \mathrm{CPC}, \mathrm{PV}\}$ can be determined.

References

1. Стратегия информатизации медицины / В.А. Лищук [и др.]. – Ейск : ЮгПолиграф, 2011. – 248 с.
2. Модель сердечно-сосудистой системы, ориентированная на современную интенсивную терапию / С.В. Фролов [и др.] // Вестн. Тамб. гос. техн. ун-та. – 2008. – Т. 14, № 4. – С. 892–902.
3. Лищук, В.А. Система закономерностей кровообращения / В.А. Лищук // Клиническая физиология кровообращения. – 2005. – № 4. – С. 14–24.
4. Лищук, В.А. Математическая теория кровообращения / В.А. Лищук. – М. : Медицина, 1991. – 256 с.
5. Лищук, В.А. Система закономерностей сердца / В.А. Лищук, Е.В. Мосткова // Клиническая физиология кровообращения. – 2006. – № 1. – С. 16–21.
6. Черырехкамерная модель сердечно-сосудистой системы / С.В. Фролов [и др.] // Вопр. соврем. науки и практики. Ун-т им. В.И. Вернадского. – 2012. – № 2(40). – С. 51–60.

Моделирование гемодинамики сердечно-сосудистой системы с пульсирующим сердцем

С.В. Фролов[1], С.В. Синдеев[1], В.А. Лищук[2], Д.Ш. Газизова[2]

Кафедра «Биомедицинская техника» (1), ФГБОУ ВПО «ТГТУ»; sergej.frolov@gmail.com; ФГБУ «Научный центр сердечно-сосудистой хирургии имени А.Н. Бакулева» РАМН (2), г. Москва

Ключевые слова и фразы: гемодинамика; математическая модель; сердечно-сосудистая система.

Аннотация: Дано математическое описание модели сердечно-сосудистой системы с пульсирующим сердцем. Сердечно-сосудистая система представлена в виде последовательно соединенных восьми упругих камер.

Modellierung der Hämodynamik des Herz-Kreislaufsystems mit dem pulsierenden Herz

Zusammenfassung: Es ist die mathematische Beschreibung des Modells des Herz-Kreislaufsystems mit dem pulsierenden Herz vorgelegt. Das Herz-Kreislaufsystem wird in der Form von acht consequent verbundenen elastischen Kamerrn dargestellt.

Modélage de la hémodynamique du système cardio-vasculaire avec un coeur pulsatoire

Résumé: Est présentée une description mathématique d'un modèle du système cardio-vasculaire avec un coeur pulsatoire. Le système cardio-vasculaire est présenté en vue de huit chambres élastiques liées successivement.

Авторы: ***Фролов Сергей Владимирович*** – доктор технических наук, профессор, заведующий кафедрой «Биомедицинская техника»; ***Синдеев Сергей Вячеславович*** – студент, ФГБОУ ВПО «ТГТУ»; ***Лищук Владимир Александрович*** – доктор биологических наук, профессор, заведующий лабораторией математического моделирования и мониторинга; ***Газизова Динара Шавкатовна*** – доктор медицинских наук, главный научный сотрудник, ФГБУ «Научный центр сердечно-сосудистой хирургии имени А.Н. Бакулева» РАМН, г. Москва.

Рецензент: ***Туголуков Евгений Николаевич*** – доктор технических наук, профессор кафедры «Техника и технологии производства нанопродуктов», ФГБОУ ВПО «ТГТУ».
