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RESEARCH METHOD FOR EXISTENCE DOMAIN OF A SOLUTION TO AN OPTIMAL CONTROL PROBLEM UNDER RANDOM PERTURBANCES

A.N. Gribkov, S.V. Artemova, I.A. Kurkin, P.A. Podkhvatilin

Department "Designing of Radio-Electronic and Microprocessor Systems", TSTU; Cbeta_Art@rambler.ru

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Abstract: The issues of studying existence domain of a solution to an optimal control problem under random perturbances such as «white» noise are considered. Mathematical apparatus of optimal control theory and method of synthesizing variables are used.

Introduction

One of the main stages of optimal control analysis is the study of issues related to the existence of a solution to an optimal control problem (**OCP**). The existence of OCP solutions and, consequently, the successful achievement of control objectives is dependent on a number of factors, which include: constraints on control actions and the control time interval [1], as well as the intensity and nature of the effect of perturbances. Perturbances are an important factor determining the construction of a control system, since they usually lead to adverse effects in the behavior of controlled objects [2], therefore it is desirable to take them into account at the early stages of optimal control system design.

The authors propose a research method for existence domain of a solution to an optimal control problem by dynamic object under random perturbances such as «white» noise.

The mathematical formulation of the optimal control problem under random perturbances

OCP is solved for object with dynamics model

$$\dot{z} = Az(t) + Bu(t) + Cw(t). \tag{1}$$

OCP is a transfer of object in set time interval $[t_0,\ t_k]$ from the initial to the final state of phase coordinates vector, i.e.

$$z(t_0) = z_0 \to z(t_k) = z_k, \tag{2}$$

with constraints on control action at each time moment

$$\forall\, t \in [t_0,\ t_k] : u(t) \in [u_l, u_h] \tag{3}$$

and minimum of the functional

$$J = f(u(t), z(t), t) \to \min. \tag{4}$$

The input data array, which is required for the numerical solution of the problem (1) – (4), is defined as

$$R = (A,\ B,\ C,\ u_l,\ u_h,\ z_0,\ z_k,\ t_0,\ t_k, \sigma_w).$$

In this problem: A, B, C – matrices of parameters for object dynamics model with $n \times n$, $n \times m$ and $n \times m$ dimensions; $z(t)$ – n-dimensional vector of phase coordinates; $u(t)$ – m-dimensional vector of control actions; $w(t)$ – m-dimensional vector of «white» Gaussian noise with zero mean values and standard deviation, which are set by m-dimensional vector σ_w; z_0, z_k – n-dimensional vectors of initial and final values of phase coordinates; u_l, u_h – m-dimensional vectors of upper and lower boundary values of the control actions; J – minimizing functional.

Description of the Research Method for Existence Domain of a Solution to an Optimal Control Problem under Random Perturbances Such as White Noise

Conditions of existence the solution of OCP under random perturbances can be determined applying the method of synthesizing variables, according to which, the normalization of the original problem is made (1) – (4) and the vector of synthesizing variables is introduced, which is uniquely determines the kind and parameters of the optimal control actions [3]. Normalization of the original problem is significantly reduces the dimension of OCP.

Research method for domain of a solution to an OCP under random perturbances such as white noise (1) – (4), contains the following steps:

1. Normalization of the original problem. In the normalized OCP, time interval and the region of acceptable values of control and disturbing factors is constant:

$$t \in [t_0; t_k] \longrightarrow T \in [0; 2];$$

$$u(t) \in [u_l; u_h] \longrightarrow U(T) \in [-1; 1];$$

$$w(t) \in [w_l; w_h] \longrightarrow W(T) \in [-1; 1].$$

It should be noted that the normalization of disturbing factors requires a priori information about the ranges of the components of the vector $w(t)$. In the case of «white» Gaussian noise with zero mean value the boundary values of perturbations can be set in accordance with the «three-sigma» rule:

$$w_l = -3\sigma_w; \quad w_h = 3\sigma_w.$$

2. Solving of normalized problem

$$Z(t_k) = \mathrm{e}^{2\overline{A}} Z(t_0) + \int_0^2 \mathrm{e}^{\overline{A}(2-\tau)} \left[\overline{B} U(\tau) + \overline{C} W(\tau)\right] d\tau,$$

where $\overline{A}$, $\overline{B}$, $\overline{C}$ – matrices of parameters for dynamics model of object in normalized problem.

3. Introduction of an n-dimensional vector of synthesizing variables Λ. In this case, equations for calculation of components of the vector Λ are derived from the solution of the normalized problem, which are obtained in the previous step:

$$\Lambda = (L_1, L_2, ..., L_n);\ \ L_i = f_i(R) = g_i(U(T), W(T), R),\ i = \overline{1, n},$$

where L_i – components of the vector Λ; f_i, g_i – functions, which are obtained from solution of the normalized problem.

4. Construction the region of existence OCP solution in the space of synthesizing variables, which is bounded by the surfaces:

$$L_j^{\lim} = g_i(U_j^{\lim}(T), W_j^{\lim}(T), R), \ i = \overline{1,n}, \ j = \overline{1,m};$$

$$U_j^{\lim}(T) = \{U_j^1, U_j^2\}; \quad U_j^1(T) = \begin{cases} -1, T < T_j^p; \\ 1, T \ge T_j^p; \end{cases} \quad U_j^2(T) = \begin{cases} 1, T < T_j^p; \\ -1, T \ge T_j^p; \end{cases}$$

$$W_j^{\lim}(T) = \{W_j^1, W_j^2\}; \quad W_j^1(T) = \begin{cases} -1, T < T_j^p; \\ 1, T \ge T_j^p; \end{cases} \quad W_j^2(T) = \begin{cases} 1, T < T_j^p; \\ -1, T \ge T_j^p; \end{cases}$$

where T_j^p – switching times.

5. Allocation of three types of areas in the space of synthesizing variables: S_g – areas of the guaranteed existence of solution; S_w – areas, where possibility of the existence of solution of OCP is determined by the random nature of the influence of noise; S_o – areas, where solution of OCP does not exist. The domain of existence OCP solution is the union of S_g and S_w

$$S = S_g \cup S_w.$$

6. Determination of the possibility of existence OCP solution depending on the position of the point $\varphi(L_1,...,L_n)$ in the region of solution existence, where coordinates of the point are values of synthesizing variables obtained in the third stage

$$L_i = f_i(R), \ i = \overline{1,n}.$$

Conditions of existence OCP solution (1) – (4) can be formulated as:

1) if $\varphi(L_1,...,L_n) \in S_g$ – solution of OCP exists;

2) if $\varphi(L_1,...,L_n) \in S_w$ – possibility of the existence OCP solution depends on the nature of noise influence;

3) if $\varphi(L_1,...,L_n) \in S_o$ – OCP solution does not exist.

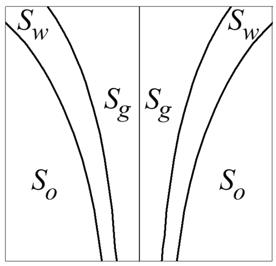
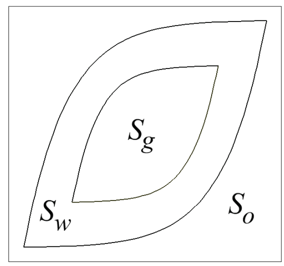
Practical application of the proposed method

The practical significance of the proposed method is that it allows checking the possibility of the existence OCP solution without determining the type of the function of optimal control directly from the values of the array of initial data components.

Graphical representation the region of solution existence for the objects described by models of dynamics in the form of an aperiodic element of the first and second orders, and also the initial data arrays are shown in Table. More detailed results of applying the proposed method can be found in [4, 5].

In the practical investigation of OCP for a specific object, the requirements related to noise stability of algorithmic support of the optimal control system can be formulated based on the results of application the research method presented in that paper. For example, if the possibility of the existence OCP solutions is defined by character of disturbing effects influence $\varphi(L_1,...,L_n) \in S_w$ (see step 6 of the method), then it is necessary to use special algorithms which allow to reduce the negative impact of noise to increase probability of achieving the control objectives.

Practical examples

<table>
<tr><td>Dynamics model of the object</td><td>$\dot{z} = az(t) + bu(t) + cw(t)$</td><td>$\begin{cases} \dot{z}_1 = z_2(t), \\ \dot{z}_2 = a_1 z_1(t) + a_2 z_2(t) + \\ \quad + bu(t) + cw(t) \end{cases}$</td></tr>
<tr><td>The input array of OCP</td><td>$R_1 = \{a, b, z_0, z_k, u_l, u_h, t_0, t_k, \sigma_w\}$</td><td>$R_2 = \{a_1, a_2, b, c, z_{10}, z_{1k}, z_{20}, z_{2k}, u_l, u_h, t_0, t_k . \sigma_w\}$</td></tr>
<tr><td>The cross sections the region of existence OCP solution in the space of synthesizing variables</td><td></td><td></td></tr>
</table>

Conclusion

The authors propose the research method for existence domain of a solution to an optimal control problem by dynamic object under random perturbances such as «white» noise.

Application of this method in the analysis phase allows checking the existence of OCP solution without defining form of optimal control function directly through the array of input data.

References

1. Грибков, А.Н. Аналитико-графический метод исследования области существования решения задачи оптимального управления многомерным объектом / А.Н. Грибков, И.А. Куркин // Информатика и системы упр. – 2011. – № 3 (29). – С. 141–152.
2. Фильтрация и стохастическое управление в динамических системах / под ред. К.Т. Леондеса. – М. : Мир, 1980. – 408 с.
3. Муромцев, Д.Ю. Методы и алгоритмы синтеза энергосберегающего управления технологическими объектами : монография / Д.Ю. Муромцев. – Тамбов ; М. ; СПб. ; Баку ; Вена : Нобелистика, 2005. – 202 с.
4. Расширенный анализ задач оптимального управления [Электронный ресурс] / С.В. Артемова [и др.] // Информ. процессы и упр. [Электронный журнал]. – 2006. – № 1. – Режим доступа : http://www.tstu.ru/ipu/2006-1/002.pdf. – Загл. с экрана.
5. Грибков А.Н. К вопросу существования решения задачи оптимального управления при наличии случайных возмущений / А.Н. Грибков // Актуальные проблемы вузовской науки: теоретические и практические аспекты : материалы Всерос. науч.-практ. конф. / Тамб. гос. ун-т. – Тамбов, 2009. – С. 226–230.

Метод исследования области существования решения задачи оптимального управления при наличии случайных возмущений

А.Н. Грибков, С.В. Артемова, И.А. Куркин, П.А. Подхватилин

Кафедра «Конструирование радиоэлектронных и микропроцессорных систем», ФГБОУ ВПО «ТГТУ»; Cbeta_Art@rambler.ru

Ключевые слова и фразы: анализ оптимального управления; «белый» шум; метод синтезирующих переменных.

Аннотация: Рассмотрены вопросы исследования области существования решения задачи оптимального управления при наличии случайных возмущений типа «белый» шум. Использован математический аппарат теории оптимального управления и метод синтезирующих переменных.

Methode der Untersuchung des Extistezraumes der Lösung der Aufgabe der Optimalsteuerung bei dem Vorhandensein der Zufallsstörungen

Zusammenfassung: Es sind die Fragen der Untersuchung des Extistezraumes der Lösung der Aufgabe der Optimalsteuerung bei dem Vorhandensein der Zufallsstörungen des Typus “weiβer” Lärm betrachtet. Es wurde den mathematischen Apparat der Theorie der Optimalsteuerung und die Methode der syntesierenden Variablen benutzt.

Méthode de l’étude du domaine de l’existence de la solution du problème de la commande optimale lors de la présence des perturbations occasionnelles

Résumé: Sont examinés les problèmes de l’étude du domaine de l’existence de la solution du problème de la commande optimale lors de la présence des perturbations occasionnelles tu type bruit «blanc». Est utilisé l’appareil mathématique de la théorie de la commande optimale et la méthode des variables synthésants.

Авторы: ***Грибков Алексей Николаевич*** – кандидат технических наук, доцент кафедры «Конструирование радиоэлектронных и микропроцессорных систем»; ***Артемова Светлана Валерьевна*** – кандидат технических наук, доцент кафедры «Конструирование радиоэлектронных и микропроцессорных систем»; ***Куркин Илья Александрович*** – аспирант кафедры «Конструирование радиоэлектронных и микропроцессорных систем»; ***Подхватилин Павел Александрович*** – студент, ФГБОУ ВПО «ТГТУ».

Рецензент: ***Майникова Нина Филипповна*** – доктор технических наук, профессор, заведующая кафедрой «Гидравлика и телотехника», ФГБОУ ВПО «ТГТУ».
